NOTE : DO NOT BREAK THE SEAL UNTIL YOU GO THROUGH THE FOLLOWING INSTRUCTIONS

COMMON ENTRANCE TEST - 2011

Question Booklet MATHEMATICS

Booklet No.

Roll No. | | | | | | |

(Enter your Roll Number in the above space)

Series A 405553

Time Allowed : 1.30 Hours **Max. Marks : 75**

INSTRUCTIONS :

1. Use only BLACK or BLUE Ball Pen.
2. All questions are COMPULSORY.
3. Check the BOOKLET thoroughly.

   IN CASE OF ANY DEFECT - MISPRINTS, MISSING QUESTION/S OR DUPLICATION OF QUESTION/S, GET THE BOOKLET CHANGED WITH THE BOOKLET OF THE SAME SERIES. NO COMPLAINT SHALL BE ENTERTAINED AFTER THE ENTRANCE TEST.
4. Before you mark the answer, fill in the particulars in the ANSWER SHEET carefully and correctly. Incomplete and incorrect particulars may result in the non-evaluation of your answer sheet by the technology.
5. Write the SERIES and BOOKLET NO. given at the TOP RIGHT HAND SIDE of the question booklet in the space provided in the answer sheet by darkening the corresponding circles.
6. Do not use any **eraser, fluid pens, blades** etc., otherwise your answer sheet is likely to be rejected whenever detected.
7. After completing the test, candidates are advised to hand over the OMR ANSWER SHEET to the Invigilator and take the candidate's copy with yourself.

SEAL

1. Let $A = \{1,2\}, B = \{\{1\},\{2\}\}, C = \{\{1\},\{1,2\}\}$. Then which of the following relation is true?

(1) $A = B$ (2) $B \subseteq C$ (3) $A \in C$ (4) $A \subset C$

2. The function $f : [0,\infty) \to [0,\infty)$ defined by $f(x) = \dfrac{2x}{1+2x}$ is :

(1) one-one and onto (2) one-one but not onto

(3) not one-one but onto (4) neither one-one nor onto

3. If $f(x) = 3 - x$, $-4 \le x \le 4$, then the domain of $\log_e(f(x))$ is :

(1) $[-4,4]$ (2) $(-\infty,3]$ (3) $(-\infty,3)$ (4) $[-4,3)$

4. If $z_r = \cos\left(\dfrac{\pi}{3^r}\right) + i\sin\left(\dfrac{\pi}{3^r}\right)$, then $z_1 \cdot z_2 \cdot z_3 \cdots$ to ∞ is equal to :

(1) -1 (2) 0 (3) $-i$ (4) i

5. If w denotes the imaginary cube roots of unity. Then the roots of the equation $(x+1)^3 + 8 = 0$ are :

(1) $-3, 1+2w, 1+2w^2$ (2) $-3, 1-2w, 1-2w^2$

(3) $-3, -1+2w, -1+2w^2$ (4) $-3, -1-2w, -1-2w^2$

6. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$, $a \ne 0$, then the relation between the coefficients of the equation is :

(1) $a^2 - b^2 + 2ac = 0$ (2) $a^2 + b^2 + 2ac = 0$

(3) $a^2 - b^2 - 2ac = 0$ (4) $a^2 + b^2 - 2ac = 0$

Space For Rough Work

7. If S_1, S_2, S_3 are the sum of $n, 2n, 3n$ terms respectively of an arithmetic progression, then :

(1) $S_3 = 2(S_1 + S_2)$ (2) $S_3 = S_1 + S_2$

(3) $S_3 = 3(S_2 - S_1)$ (4) $S_3 = 3(S_2 + S_1)$

8. If $4^x = 16^y = 64^z$, then :

(1) x, y, z are in G.P. (2) x, y, z are in A.P

(3) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in G.P (4) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P

9. The coefficient of $p^n q^n$ in the expansion of $[(1+p)(1+q)(p+q)]^n$ is :

(1) $\sum_{k=0}^{n} [C(n,k)]^2$ (2) $\sum_{k=0}^{n} [C(n,k+2)]^2$ (3) $\sum_{k=0}^{n} [C(n,k+3)]^2$ (4) $\sum_{k=0}^{n} [C(n,k)]^3$

10. The constant terms is the expansion of $\left(\sqrt{x} - \frac{c}{x^2}\right)^{10}$ is 180, then the value of c equals to :

(1) ± 2 (2) ± 3 (3) ± 4 (4) None of these

11. The number of even numbers of three digits which can be formed with digits $0,1,2,3,4$ and 5 (no digit being used more than once) is :

(1) 60 (2) 92 (3) 52 (4) 48

12. A student is allowed to select at most n-books from a collection of $(2n+1)$ books. If the total number of ways in which he can select a book is 255, then the value of n equals to :

(1) 6 (2) 5 (3) 4 (4) 3

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13. The value of $\sum_{n=0}^{\infty} \frac{n^2+4}{n!}$ is equal to :

(1) $6e$ (2) $5e$ (3) $4e$ (4) None of these

14. If the values observed are 1, 2,3,...,n each with frequency 1 and n is even, then the mean deviation from mean equals to :

(1) n (2) $\frac{n}{2}$ (3) $\frac{n}{4}$ (4) None of these

15. The three distinct points $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$ and $C(0,a)$ (where a is a real number) are collinear if :

(1) $t_1t_2 = -1$ (2) $t_1t_2 = 1$ (3) $2t_1t_2 = t_1 + t_2$ (4) $t_1 + t_2 = a$

16. A line segment of 8 units in length moves so that its end points are always on the co-ordinate axes. Then, the equation of locus of its midpoint is :

(1) $x^2 + y^2 = 4$ (2) $x^2 + y^2 = 16$ (3) $x^2 + y^2 = 8$ (4) $|x| + |y| = 8$

17. The equation of the line passing through $(0,0)$ and intersection of $3x - 4y = 2$ and $x + 2y = -4$ is :

(1) $7x = 6y$ (2) $6x = 7y$ (3) $5x = 8y$ (4) $x = 0$

18. The number of straight lines which can be drawn through the point $(-2,2)$ so that its distance from $(3,-1)$ will be equal 6 units is :

(1) One (2) Two (3) Infinite (4) Zero

19. The value of k for which the equation $x^2 - 4xy - y^2 + 6x + 2y + k = 0$ represents a pair of straight lines is :

(1) $k = 4$ (2) $k = -1$ (3) $k = \frac{-4}{5}$ (4) $k = \frac{-22}{5}$

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20. Suppose the straight line $x + y = 5$ touches the circle $x^2 + y^2 - 2x - 4y + 3 = 0$. Then, the co-ordinates of the point of contact are :

(1) (3,2) (2) (2,3) (3) (4,1) (4) (1,4)

21. If the straight line $y = 2x + c$ is a tangent to the ellipse $\frac{x^2}{3} + \frac{y^2}{4} = 1$, then c equals to :

(1) ± 4 (2) ± 6 (3) ± 8 (4) ± 1

22. $f(x) = |\sin 2x| + |\cos 2x|$ is a periodic function with period :

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

23. The value of $\cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{4\pi}{7}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{-1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{-1}{8}$

24. Let C be right-angle of a triangle ABC, then $\frac{\sin^2 A}{\sin^2 B} - \frac{\cos^2 A}{\cos^2 B}$ is equal to :

(1) $\frac{a^2 - b^2}{ab}$ (2) $\frac{a^4 - b^4}{a^2 b^2}$ (3) $\frac{a^4 + b^4}{a^2 b^2}$ (4) $\frac{a^2 + b^2}{ab}$

25. The rational number among the following real numbers is :

(1) $\sin 15^\circ$ (2) $\cos 15^\circ$ (3) $\sin 15^\circ \cdot \cos 15^\circ$ (4) $\sin 15^\circ \cdot \cos 75^\circ$

26. In a triangle $ABC, a = 8\,\text{cm}$, $b = 10\,\text{cm}$, $c = 12\,\text{cm}$. The relation between angles of the triangle is :

(1) $C = A + B$ (2) $C = 2B$ (3) $C = 2A$ (4) $C = 3A$

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27. The general solution of $\cos x \cdot \cos 6x = -1$ is :

(1) $x = (2n+1)\frac{\pi}{7}, n \in \mathbb{Z}$ (2) $x = (2n+1)\frac{\pi}{5}, n \in \mathbb{Z}$

(3) $x = (2n+1)\frac{\pi}{35}, n \in \mathbb{Z}$ (4) $x = (2n+1)\pi, n \in \mathbb{Z}$

28. The number of solutions of the equation $\sin x + \sin 5x = \sin 3x$ lying in the interval $[0, \pi]$ is :

(1) 4 (2) 6 (3) 5 (4) 2

29. The value of $\tan\left[\cos^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is :

(1) 6 (2) $\frac{17}{6}$ (3) $\frac{6}{17}$ (4) None of these

30. The value of $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$ is :

(1) 1 (2) 0 (3) -1 (4) $\frac{\pi}{2}$

31. If A is a square matrix of order 3 and α is a real number, then determinant $|\alpha A|$ is equal to :

(1) $\alpha^2|A|$ (2) $\alpha|A|$ (3) $\alpha^3|A|$ (4) None of these

32. The system of equations $2x - y + z = 0$, $ax - y + 2z = 0$, $x - 2y + z = 0$ has non-zero solution if α is equal to :

(1) 1 (2) 2 (3) 4 (4) 5

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33. The area of the triangle whose vertices are the points $(a(a+1), a+1)$, $((a+1)(a+2), a+2)$ and $((a+2)(a+3), a+3)$ is equal to :

(1) -1 (2) 1

(3) $\frac{1}{2}$ (4) $a(a+1)(a+2)(a+3)$

34. The matrix product satisfies $[5 \quad 6 \quad 2] \cdot A^T = [4 \quad 8 \quad 1 \quad 7 \quad 8]$, where A^T denotes the transpose of the matrix A. Then, the order of the matrix A equals to :

(1) 1×2 (2) 5×1 (3) 3×5 (4) 5×3

35. If A is a 2×2 matrix and $|A| = 2$, then the matrix represented by $A(adj\, A)$ is equal to :

(1) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (3) $\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$ (4) $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

36. Let A and B both be 3×3 matrices. Then, $(AB)^T = BA$ if

(1) A is skew-symmetric and B is symmetric

(2) B is skew-symmetric and A is symmetric

(3) A and B are skew-symmetric

(4) None of these

37. Let $\vec{a}$ and $\vec{b}$ be the position vector of A and B respectively. The position vector of a point C on $\overrightarrow{AB}$ produced such that $\overrightarrow{AC} = 4\,\overrightarrow{AB}$ is equal to :

(1) $\frac{4\vec{b} - \vec{a}}{3}$ (2) $4\vec{b} - 3\vec{a}$ (3) $4\vec{a} - 3\vec{b}$ (4) $\frac{4\vec{a} - \vec{b}}{3}$

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38. The sum of two vectors $\vec{a}$ and $\vec{b}$ is a vector $\vec{c}$ such that $|\vec{a}| = |\vec{b}| = |\vec{c}| = 2$. Then the magnitude of $\vec{a} - \vec{b}$ is equal to :

(1) $\sqrt{3}$ (2) 2 (3) $2\sqrt{3}$ (4) 0

39. The position vector of two given points A and B are $4\vec{i} - 3\vec{j} - \vec{k}$ and $5\vec{i} - 5\vec{j} + \vec{k}$ respectively. If γ is the angle between $\overrightarrow{AB}$ and Z-axis, then $\cos\gamma$ is equal to :

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{-2}{3}$ (4) 0

40. The vectors $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} + 2\vec{j} - 3\vec{k}$ and $3\vec{i} + \lambda\vec{j} + 5\vec{k}$ are co-planar if λ equals to :

(1) 1 (2) -1 (3) -4 (4) 4

41. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the unit vectors such that $\vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} = 0$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then $\vec{c}$ equals to :

(1) $\pm\frac{2}{\sqrt{3}}(\vec{a} \times \vec{b})$ (2) $\pm\frac{\sqrt{3}}{2}(\vec{a} \times \vec{b})$

(3) $\pm 2(\vec{a} \times \vec{b})$ (4) $\pm\frac{1}{2}(\vec{a} \times \vec{b})$

42. For the non-zero vectors, $\vec{a}, \vec{b}$ and $\vec{c}$, the relation $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$ is true if :

(1) $\vec{b} \perp \vec{c}$ (2) $\vec{a} \perp \vec{b}$ (3) $\vec{a} \parallel \vec{c}$ (4) $\vec{a} \perp \vec{c}$

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43. The ratio in which ZX-plane divides the line segment AB joining the points $A(4,2,3)$ and $B(-2,4,5)$ is equal to :

(1) 1 : 2 internally
(2) 1 : 2 externally
(3) $-2:1$
(4) None of these

44. The number of lines making equal angles with the co-ordinate axes in three dimensional geometry is equal to :

(1) 3
(2) 4
(3) 2
(4) None of these

45. The projection of a line segment OP through origin O, on the co-ordinate axes are 8, 5, 6. Then the length of the line segment OP is equal to :

(1) 5
(2) $5\sqrt{5}$
(3) $10\sqrt{5}$
(4) None of these

46. Suppose $P(2,y,z)$ lies on the line through $A(3,-1,4)$ and $B(-4,2,1)$. Then, the value of z is equal to :

(1) $\frac{-1}{2}$
(2) $\frac{19}{4}$
(3) $\frac{-19}{4}$
(4) $\frac{25}{7}$

47. The length of the perpendicular distance of the point $(-1,4,0)$ from the line $\frac{x}{1}=\frac{y}{3}=\frac{z}{1}$ is equal to :

(1) $\sqrt{6}$
(2) $\sqrt{5}$
(3) 2
(4) 1

48. The equation of the plane perpendicular to the Z-axis and passing through $(2,-3,5)$ is :

(1) $x-2=0$
(2) $y+3=0$
(3) $z-5=0$
(4) $2x-3y+5z+4=0$

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49. The plane $2x - 2y + z + 5 = 0$ is a tangent to the sphere $(x-2)^2 + (y-2)^2 + (z-1)^2 = r^2$ if r equals :

(1) 1 (2) 2 (3) 4 (4) None of these

50. The contrapositive statement of the proposition $p \to \sim q$ is :

(1) $\sim p \to q$ (2) $\sim q \to p$ (3) $q \to \sim p$ (4) None of these

51. A bag has four pair of balls of four distinct colours. If four balls are picked at random (without replacement), the probability that there is at least one pair among them have the same colour is :

(1) $\frac{1}{7!}$ (2) $\frac{8}{35}$ (3) $\frac{19}{35}$ (4) $\frac{27}{35}$

52. If A and B are mutually exclusive events such that $P(A) = 0.25$, $P(B) = 0.4$, then $P(A^c \cap B^c)$ is equal to :

(1) 0.45 (2) 0.55 (3) 0.9 (4) 0.35

53. Suppose $f(x) = \frac{k}{2^x}$ is a probability distribution of a random variable X that can take on the values $x = 0, 1, 2, 3, 4$. Then k is equal to :

(1) $\frac{16}{15}$ (2) $\frac{15}{16}$ (3) $\frac{31}{16}$ (4) None of these

54. Let $f(x) = \frac{3}{1 + 3^{\tan x}}$. Then which of the following is true?

(1) $\lim_{x \to \frac{\pi}{2}^-} f(x) = 3$ (2) $\lim_{x \to \frac{\pi}{2}^+} f(x) = 0$ (3) $\lim_{x \to \frac{\pi}{2}^+} f(x) = 3$ (4) $\lim_{x \to \frac{\pi}{2}} f(x)$ exists

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55. If $f(x)=\dfrac{1}{2-x}$, then $f(f(x))$ is discontinuous at :

(1) $x=2,4$ (2) $x=4,\dfrac{3}{2}$ (3) $x=2,\dfrac{3}{2}$ (4) $x=4$

56. If $f(x)=|x-2|\log_{10}(x-1)$, then f is differentiable in :

(1) $\mathbb{R}-\{1,11\}$ (2) $\mathbb{R}-\{2,11\}$ (3) $\mathbb{R}-\{11\}$ (4) $\mathbb{R}-\{1,2\}$

57. If $g(x)$ is the inverse of $f(x)$ and $f'(x)=\cos x$, then $g'(x)=$

(1) $\sec x$ (2) $\sec(g(x))$ (3) $\cos(g(x))$ (4) $-\sin(g(x))$

58. If $x\neq 0$ and $y=\log_e|2x|$, then $\dfrac{dy}{dx}=$

(1) $\dfrac{1}{x}$ (2) $\dfrac{-1}{x}$ (3) $\pm\dfrac{1}{2x}$ (4) None of these

59. The derivative of $\operatorname{cosec}^{-1}\left(\dfrac{1}{2x\sqrt{1-x^2}}\right)$ with respect to $\sqrt{1-x^2}$ is :

(1) $\dfrac{1}{\sqrt{1-x^2}}$ (2) $\dfrac{2}{x}$ (3) $\dfrac{-2}{x}$ (4) $\dfrac{-1}{\sqrt{1-x^2}}$

60. The curves $\dfrac{x^2}{a^2}+\dfrac{y^2}{2}=1$ and $y^2=8x$ intersect at right angles if a^2 is equal to :

(1) $\dfrac{1}{2}$ (2) 1 (3) 2 (4) None of these

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61. If $f(x) = kx - \cos x$ is monotonically increasing for all $x \in \mathbb{R}$, then

(1) $k > -1$ (2) $k < 1$ (3) $k > 1$ (4) None of these

62. If $f(x) = 2x^2 - |x| + 4,\ x \in [-1,2]$. Then, for some $c \in (-1,2)$, $f'(c) =$

(1) $\dfrac{f(2) - f(0)}{2 - 0}$ (2) $\dfrac{f(2) - f(-1)}{2 - (-1)}$

(3) $\dfrac{f(1) - f(-1)}{1 - (-1)}$ (4) None of these

63. The value of the integral $\int \dfrac{1}{e^{2x} + e^{-2x}}\, dx$ is equal to :

(1) $2\tan^{-1}(e^{2x}) + c$ (2) $\tan^{-1}(e^{2x}) + c$

(3) $\dfrac{1}{2}\tan^{-1}(e^{2x}) + c$ (4) $\dfrac{-1}{(e^{2x} + e^{-2x})^2} + c$

64. The value of the integral $\int \dfrac{-xe^x}{(x+1)^2}\, dx$ is equal to :

(1) $\dfrac{-e^x}{(x+1)^2} + c$ (2) $\dfrac{e^x}{(x+1)^2} + c$ (3) $\dfrac{e^x}{(x+1)} + c$ (4) $\dfrac{-e^x}{x+1} + c$

65. The value of $\int_4^8 \dfrac{\sqrt{x}}{\sqrt{x} + \sqrt{12 - x}}\, dx$ is equal to :

(1) 4 (2) 2 (3) 1 (4) $\dfrac{1}{2}$

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66. $\int_{-3}^{3}[f(x)+f(-x)]\cdot[g(x)-g(-x)]dx$ is equal to :

(1) 0 (2) $2\int_{-3}^{3}f(x)\,dx$ (3) $2\int_{0}^{3}f(x)g(x)dx$ (4) $2\int_{0}^{3}[f(x)-g(x)]dx$

67. The area bounded by the curve $y=1+\log_e x$, the x-axis and the straight line $x=e$ is equal to (in square units) :

(1) $3e-2$ (2) e (3) $e-\frac{1}{e}$ (4) $e+\frac{1}{e}$

68. The degree of the differential equation $\frac{d^2y}{dx^2}=\frac{5y+\frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$ is equal to :

(1) 2 (2) 3 (3) 4 (4) $\frac{5}{2}$

69. The general solution of the differential equation $\frac{d^2y}{dx^2}=e^{2x}+e^{-x}$ is :

(1) $4e^{2x}+e^{-x}+c_1x+c_2$ (2) $\frac{1}{4}e^{2x}-e^{-x}+c$

(3) $\frac{1}{4}e^{2x}+e^{-x}+c_1x+c_2$ (4) $\frac{1}{4}e^{2x}-e^{-x}+c_1x+c_2$

70. Three forces P,Q and R acting at a point O in the plane. The measure of angles $\angle POQ$ and $\angle QOR$ are 120° and 90° respectively. Then, the equilibrium forces P,Q and R are in the ratio :

(1) $3:1:2$ (2) $2:1:3$ (3) $\sqrt{3}:1:2$ (4) $2:1:\sqrt{3}$

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71. The maximum and minimum magnitude of resultants of two forces are P_1 and P_2 respectively. The magnitude of the resultant when two forces are at right angles is equal to :

(1) $2\sqrt{P_1P_2}$ (2) $\sqrt{P_1^2 + P_2^2}$

(3) $\frac{\sqrt{P_1^2 + P_2^2}}{2}$ (4) $\sqrt{\frac{P_1^2 + P_2^2}{2}}$

72. A force 2 unit acts along the line $x - 4 = y - 5$. The moment of the force about the point $(1, 1)$ along Z-axis is equal to :

(1) 0 (2) $\frac{1}{\sqrt{2}}$ (3) $\sqrt{2}$ (4) $2\sqrt{2}$

73. The distance travelled by a bus in t-seconds after the brakes are applied is $1 + 2t - 2t^2$ meters. The distance travelled by the bus before it stops is equal to :

(1) 0.5 meters (2) 1 meter (3) 1.5 meters (4) 2.5 meters

74. A particle is thrown vertically upwards with velocity 24.5 cm per minute. It will return to the original position after :

(1) 1 second (2) 3 seconds (3) 1.5 seconds (4) None of these

75. Two balls are projected from the same point in directions inclined at 45° and 60° to the horizontal respectively. If they attain the same height, the ratio of their velocities of projection is equal to :

(1) $\sqrt{3} : 1$ (2) $3 : 1$ (3) $3 : 2$ (4) $\sqrt{3} : \sqrt{2}$

Space For Rough Work

Space For Rough Work